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Simulation of The Spread of Covid-19 Disease Considering The Vaccination Factor

Tasmi[1], K. Adam[2], N. Aliya[3], N. Selvia[4], Y. Putri[5], Berliani[6], M. Z. Almuzakki[7]

Department of Computer Science, Universitas Pertamina, Jakarta 12220, Indonesia
*Corresponding Author: tasmi@universitaspertamina.ac.id
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Abstracts

Since the end of 2019, COVID-19 has been causing a worldwide pandemic and changing several aspects of human life, mainly Indonesian. The emergence of vaccines was a significant milestone in the process of solving the pandemic worldwide. The vaccine consists of several stages. The rate of vaccine administration at each stage is very important to know its influence on changes in the number of people infected with the vaccine. In this work, we develop a SIR-based mathematical model with additional multi-staged vaccination and death compartments to study the effect of a multi-staged vaccination scheme in Indonesia. The model is analyzed numerically using Python programming language by using Python's library to solve differential equations, namely scpiy.integrate.odeint. Furthermore, sensitivity analysis related to the multi-staged vaccination parameters is considered. The results show that broader implementation of vaccination significantly lowers the number of actively infected individuals.

Keywords: COVID-19**,** mathematical model**,** staged vaccination scheme**,** sensitivity analysis

Introduction

At the end of 2019, the World Health Organization (WHO) announced a new virus that spreads from human to human called the SARS-CoV-2 which causes an illness called COVID-19. SARS-CoV-2 is a virus that attacks human's respiratory system like flu and more serious diseases such as SARS. According to [1], an individual infected with COVID-19 has several symptoms ranging from coughing and fever to experiencing respiratory problems. In addition, extensive pneumonia infiltrate is identified from the X-ray results of both lungs. In some severe cases, it can cause death.

Based on data in [3], 6,776,984 confirmed cases of Covid-19 were reported, including 14,205 active cases, 161,327 deaths, and 1,254 cases identified with Covid-19 symptoms. The number of COVID-19 cases, which continues to increase daily, has caused the Indonesian government to implement several preventive measures: lockdown, PPKM, and vaccination. At that time, one of the steps taken by the government was to provide vaccines to the Indonesian population.

Vaccination is a liquid that is injected into the human body. Vaccines aim to increase the human immune system against a disease. If a person is infected with a disease, the risk posed by that disease will not hurt the body. Vaccination against COVID-19 is one way to prevent the spread of COVID-19 and build herd immunity. If the vaccination rate is low, it can result in the body of someone who has not been vaccinated being susceptible to the Covid-19 virus. The implementation of the COVID-19 vaccination in Indonesia is being carried out in stages. Based on data taken from [2], the vaccine target in Indonesia reached 234,666,020 inhabitants, with the number of people undergoing the first stage of COVID-19 vaccination being 203,831,792 inhabitants, the second stage of vaccination being 106,131,744 inhabitants, the third phase of vaccination was 68,739,017 inhabitants. One way that can be used to find out how much influence vaccination has on reducing COVID-19 cases is through mathematical modelling.

Previous research has carried out modeling of the spread of COVID-19 cases. [4] carried out an analysis of the COVID-19 model using the Susceptible-Infected-Recovered (SIR) approach by randomly adding vaccination factors to newborn individuals, thus becoming a Susceptible-Vaccinated-Infected-Recovered (SVIR) model. Furthermore, there is also a model built previously, namely the delay model, by considering vaccination factors, which develops the basic SEIR model [5]. The same basic model is also used in [6]; the model considers vaccination and isolation factors as model parameters. Furthermore, from this model, a numerical simulation was carried out using secondary data for COVID-19 cases in Indonesia. [7] developed a transmission model of COVID-19 in Nigeria. The model accommodates awareness programs and inpatient treatment strategies for mild and severe cases. In [8] build a COVID-19 model considering vaccination and quarantine in Kerala Indians. The model in [9] is a fractional COVID-19 model considering vaccines. The model is solved using a new numerical scheme that modifies Simpson's 1/3 method. Many other similar models have also been developed, including [10], [11], [12], [13], [14], [15], [16], and [17]. In this research, the basic model used is the SIR Model.

[18] built the SIR model to overcome the problem of the spread of infectious diseases, for example, the COVID-19 virus, which is discussed in this paper. The influence of vaccination expands Kermack and McKendrik's SIR model. On the other hand, handling the spread of COVID-19 involves vaccination, so paying attention to the vaccination factor in modeling is necessary. Thus, this paper studies the spread of COVID-19 using the SIR model and involving vaccination factors. As is known, the Covid-19 vaccination consists of several stages. In this study, vaccination factors will be accommodated, including stage 1, stage 2, and stage 3 (Booster Vaccine), but this study does not consider the type of vaccine (Sinovac or others). This research aims to determine the effect of the rate of vaccine administration at each stage on reducing COVID-19 cases.

In general, this paper contains a model for the spread of the COVID-19 disease by considering three stages of vaccination using the basic model of disease spread, namely SIR. Next, from the model obtained, the solution is sought numerically, and a sensitivity analysis is carried out on the vaccination rate parameters at each stage. From the results of this sensitivity analysis, the effect of vaccination on reducing COVID-19 cases is evident.

Material and Methods

This research begins by identifying the problem that arises, namely the problem of the spread of the COVID-19 disease; one way to overcome it is by using vaccination. A literary study of several related references was also carried out. After that, building model assumptions are close to actual conditions. From these assumptions, a compartment model (compartment diagram) is drawn to become the basis for building the model. The model built is a system of differential equations which is solved numerically using the built-in Python library using secondary data obtained from various sources and data assumptions. Sensitivity analysis of vaccination parameters was carried out by trying several vaccination rate values at each vaccination stage. The sensitivity analysis result is used to see the effect of vaccines on reducing COVID-19 cases. The numerical solution will be analyzed and interpreted so that numerical conclusions will be drawn from the model that has been built.

Mathematical Model

In general, the spread of disease can be modeled using the basic model of disease spread, namely the SIR model with S(Susceptible), I(Infective), and R(Recovered). Researchers have previously developed many models of disease spread based on the SIR model. In this study, the SIR model was used as a basic model to develop a model for the spread of the COVID-19 disease. The SIR model was developed by adding cases of death due to disease and a vaccination factor, which consists of 3 stages: vaccine stage 1, vaccine stage 2, and vaccine stage 3 (Booster). The compartments added to this study were the compartment for the group of individuals who had the first vaccine (V_1), the compartment for the group of individuals who had the second vaccine (V_2), and the compartment for the group of individuals who had the third vaccine (V_3), and the compartment that died due to disease (D).

In detail, the assumptions used to build the disease spread model are as follows:

1. Individuals born are considered healthy and have a birth rate of α. Furthermore, healthy individuals have a natural death rate of μ. Healthy individuals are individuals who are susceptible and have the potential to be exposed to and infected with COVID-19 at a rate of β. All susceptible individuals can do the first phase of the vaccine. The administration rate of the vaccine's first phase to susceptible individuals is γ_1.
2. Individuals vaccinated in the first stage will enter compartment V_1. Individuals who have already had the first phase of the vaccine can also be infected with COVID-19 at a rate of β_1. Individuals vaccinated with one have a natural death rate of μ. Then, individuals vaccinated for stage one will be continued with the vaccine for stage two at a rate of γ_2.
3. Compartment V_2 represents a group of individuals vaccinated with stage two. Individuals in this compartment have a natural mortality rate of μ. Individuals in the compartment have a disease infection rate of β_3. However, individuals in this compartment can also proceed to the next stage of the vaccine at a rate of γ_3.
4. Individuals infected with COVID-19 and declared infected with COVID-19 will enter the Infected compartment with the notation, I. Individuals in this compartment can die naturally at a rate, μ, and from the disease at a rate, ρ, with probability of dying due to illness amounting to p. However, infected individuals can also recover at a rate of ω.
5. Individuals who have recovered from COVID-19 are grouped in compartment R. In this compartment, individuals die naturally at the same rate as the rate of natural death in other compartments.

The flow of the spread of COVID-19 based on the assumptions that have been built is illustrated in a compartment diagram presented in the following figure:

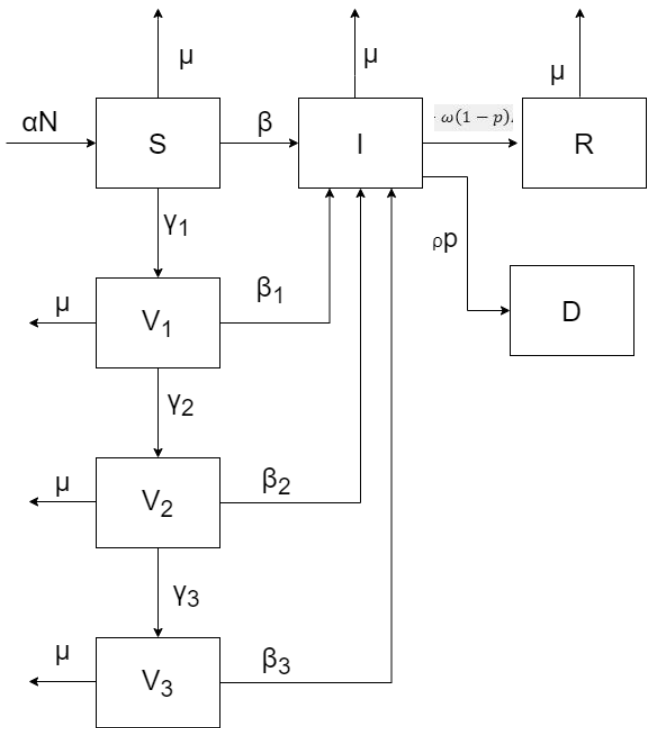

Figure 1. Flow diagram of the Covid-19 Spread Model

Based on the compartment diagram, the disease spread model is obtained as follows:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \alpha N - \frac{\beta SI}{N} - \gamma_1 S - \mu S \\
\frac{dI}{dt} &= \frac{\beta SI}{N} + \frac{\beta_1 V_1 I}{N} + \frac{\beta_2 V_2 I}{N} + \frac{\beta_3 V_3 I}{N} - \rho p I - \omega(1-p)I - \mu I \\
\frac{dR}{dt} &= \omega(1-p)I - \mu R \\
\frac{dD}{dt} &= \rho p I \\
\frac{dV_1}{dt} &= \gamma_1 S - \frac{\beta_1 V_1 I}{N} - \gamma_2 V_1 - \mu V_1 \\
\frac{dV_2}{dt} &= \gamma_2 V_1 - \frac{\beta_2 V_2 I}{N} - \gamma_3 V_2 - \mu V_2 \\
\frac{dV_3}{dt} &= \gamma_3 V_2 - \frac{\beta_3 V_3 I}{N} - \mu V_3
\end{aligned}\right\} \quad (1)$$

Information on the variables and parameters used in the mathematical model in the system of equation (1) above is presented in the table below:

Table 1. Description of SIR model variables and parameters

Symbol	Description	Value
S	Number of Susceptible	People
I	Number of *Infection*	People
R	Number of *Recovered*	People
D	The number of people who died from COVID-19	People
N	Total population $S + I + R + V_1 + V_2 + V_3$	People
V_1	Number of individuals who received the first vaccine	People
V_2	Number of individuals who did the second vaccine	People
V_3	Number of individuals who did the third vaccine	People
t	Time	Day
M	Natural death rate	1/day
γ_1	Level 1 vaccination rate	1/day
γ_2	Level 2 vaccination rate	1/day
γ_3	Level 3 vaccination rate	1/day
β	The rate at which susceptible individuals are exposed to the virus	1/day
β_1	The rate at which individuals vaccinated in the first phase become exposed to the virus	1/day

Symbol	Description	Value
β_2	The rate of second stage vaccinated individuals becoming exposed to the virus	1/day
β_3	The rate at which individuals vaccinated in the third phase become exposed to the virus	1/day
α	Natural birth rate	1/day
ω	Natural healing rate	1/day
ρ	Death rate due to Covid-19	1/day
P	Probability of Covid-19 sufferers dying	-

Results and Discussion

The model for the spread of COVID-19 presented in the system of equations (1) is solved numerically using the Python programming language that utilizes the scipy.integrate.odeint library.,

Case Study I
In the first case study, a numerical simulation was carried out using the data in the table below.

Table 2. Description of variables and parameters of the SIR model

Symbol	Value	*Source*	Unit
$S(0)$	65,165,024	-	People
$I(0)$	14,205	[3]	People
$R(0)$	6,601,452	[3]	People
$D(0)$	161,327	[3]	People
$V_1(0)$	28,961,031	[3]	People
$V_2(0)$	106,131,744	[3]	People
$V_3(0)$	68,739,017	[3]	People
N	275,773,800	[2]	People
μ	0.00001	Assumed	1/day
γ_1	0.0008	Assumed	1/day
γ_2	0.0009	Assumed	1/day
γ_3	0.0004	Assumed	1/day
β	0.5	Assumed	1/day
β_1	0.333 β	Assumed	1/day
β_2	0.333β_1	Assumed	1/day
β_3	0.333β_2	Assumed	1/day
α	0.00001	[19]	1/day
ω	0.09	Assumed	1/day
ρ	0.002	Assumed	1/day
p	0.15	Assumed	

By using Table 2, the numerical solution of the model that has been built is presented in Figure 2 until Figure 8. The data in Table 2 is Indonesian data, namely accumulated COVID-19 data from the beginning to May 2023. Figure 2 represents population dynamics for susceptible, infected, recovered, and dead compartments. In the figure, it can be seen that after day 110, the number of people who recovered exceeded the number of susceptible people. This phenomenon is expected because the total population reached 275,773,800 and the number of people who recovered; if seen in the compartment model in Figure 1, the number of people who recovered came from infected people, while infected people enter from various compartments (susceptible, vaccinated 1, 2, and 3). Each of these compartments has a high initial value (The initial values taken can be seen in Table 2).

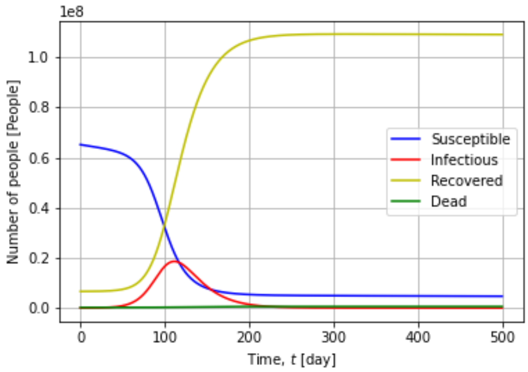

Figure 2. Number of people in each compartment at any time

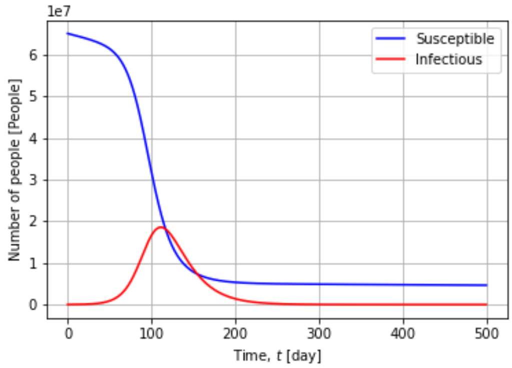

Figure 3. Number of people in the susceptible and infectious compartments

Furthermore, a picture of the relationship between populations and compartments is presented to make it easier to see the relationship between the two compartments. Susceptible and infected compartments/populations are presented in Figure 3. The figure shows that initially, there were around 65 million people in the susceptible compartment. Over time, several people in that compartment experienced quite drastic transmission around the first 100 days. Several factors affect this compartment: a reduction in the population going to the vaccine and infection compartments. However, this decline over time reaches a constant number of around 150 days, so that over time, there are around 8 million left. This means that the rest of the people here are not exposed to COVID-19 and are not vaccinated. Along with the dynamics of the population in the susceptible compartment, in Figure 3, it appears that people infected with COVID-19 also experienced an increase around the first hundred days. The infected population peaked around day 110, then decreased to zero.

In Figure 4, it can be seen that the population in vaccine compartment one has increased. However, over time, it has decreased because, in this population, there are infected people and people who continue to receive vaccine 2. From Figure 4, it can be seen that around the first hundred days, the number decreases. The population in the susceptible compartment was more drastic than the decrease in the number of people in the vaccination population. After that, the population in vaccine compartment 1 was more significant than in the susceptible compartment.

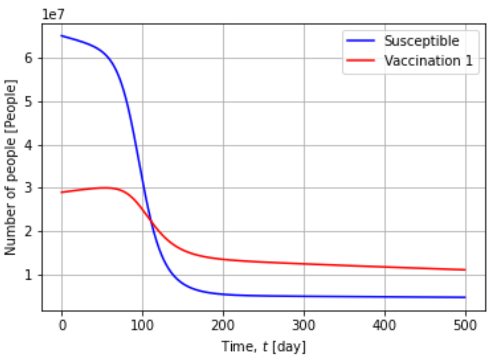

Figure 4. Number of people in the susceptible and vaccine compartment 1

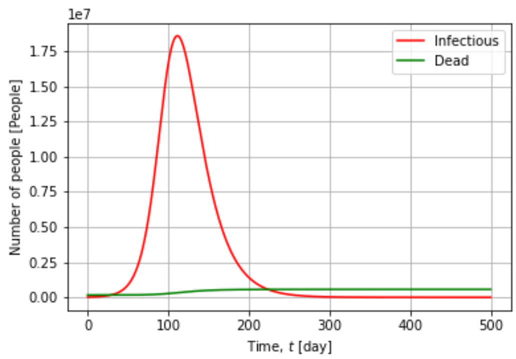

Figure 5. Number of people who were in the infection compartment and died

Figure 5 shows the number of people infected and who died from COVID-19. The number of people who died from Covid-19 is far less than that of those who died from Covid-19. However, as time passes, around the 200th day, the number of infected people decreases. This reduction is caused by those with the infection who have recovered or died. In Figure 6, people who recovered from COVID-19 tended to dominate compared to infected people and those who died. This phenomenon occurs because the initial values for each compartment are not taken at the start of COVID-19, but it is assumed that COVID-19 has already occurred. Hence, people who recovered from COVID-19 at the start are people who recovered from Covid in the previous period.

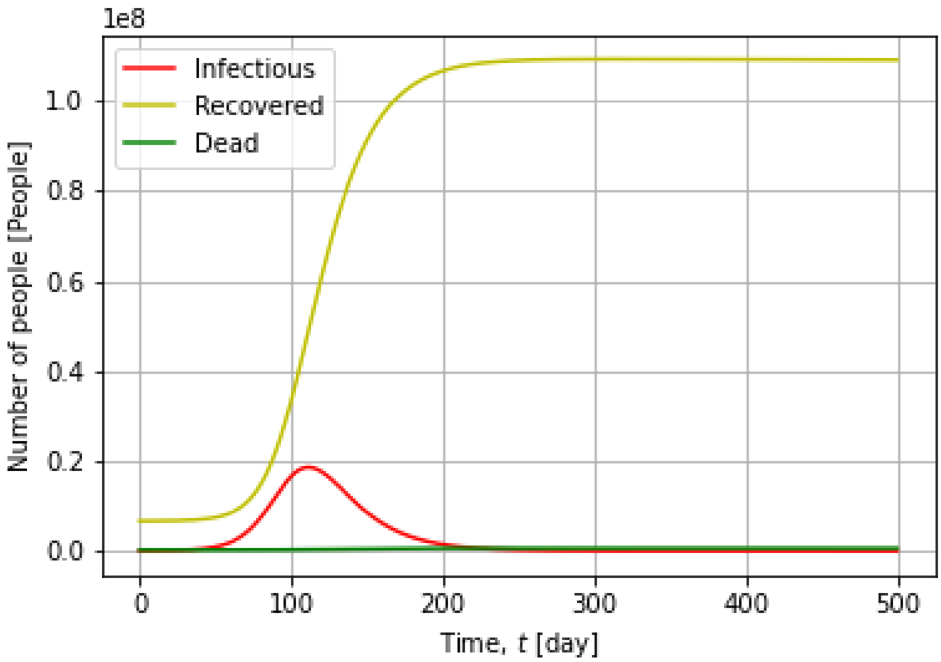

Figure 6. Several people who were in the infection compartment recovered and died

Figure 7 shows the number of people who have received two and three vaccines within 500 days. In the figure, the number of people who get vaccine 3 is much lower than people who get vaccine three. This occurrence is because the initial value for vaccine 2 is higher than that for vaccine 3 (based on the data used in Table 2). Apart from that, the vaccination rate from vaccines 2 to 3 is also much lower than that from vaccines 1 to 2.

Figure 8 is a picture of the number of people at any time for people who are on Vaccine 1 and Vaccine 2. The picture shows that the number of people vaccinated twice daily has decreased because people in that compartment moved to vaccine compartment 3. Around day 350, the number of people who were vaccinated 3 exceeded the number of people in vaccine compartment 2. For Vaccine Compartment 1 and Vaccine 2, the number of people in the two compartments decreased over time. However, the number of people in Compartment 1 was the highest between Vaccine Compartment 2 and Vaccine 3. This phenomenon shows that not all people vaccinated for the first time did not get vaccines 2 and 3.

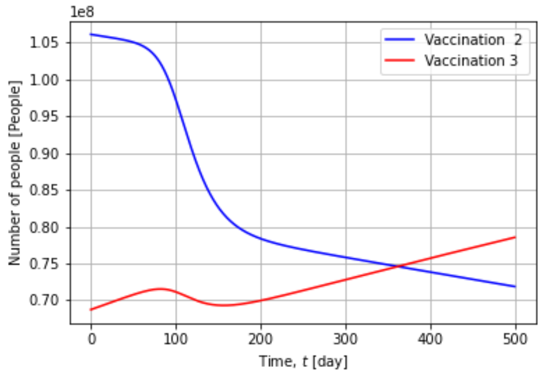

Figure 7. Number of people in Vaccine Compartment 2 and Vaccine 3

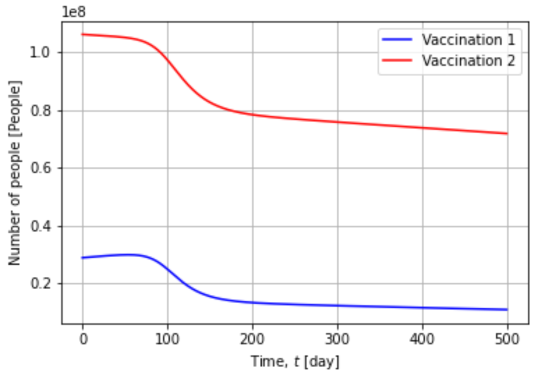

Figure 8. The number of people in Vaccine Compartment One and vaccine 2

Case Study 2: Sensitivity of Vaccination Parameters

In this section, sensitivity to vaccination parameters is carried out. Sensitivity analysis was done because, in this study, no parameter estimation was carried out from the original data, so parameter sensitivity was carried out to determine the influence of the parameters on the dynamics of the variables in the model. Data on the sensitivity of vaccination parameters are presented in the table below.

Sensitivity of vaccination parameters, γ_1

Symbol	Value	*Source*	Unit
γ_1	0.0008	Assumed	1/day
γ_1	0.008	Assumed	1/day
γ_1	0.08	Assumed	1/day

The dynamics of the variables presented in the figure below are obtained. The focus of vaccination is to see whether the infected and those who die will change, so the pictures presented in this section are the dynamics of the vaccination populations 1, 2, and 3; infected population dynamics; population recovered from COVID-19; population died causes COVID-19.

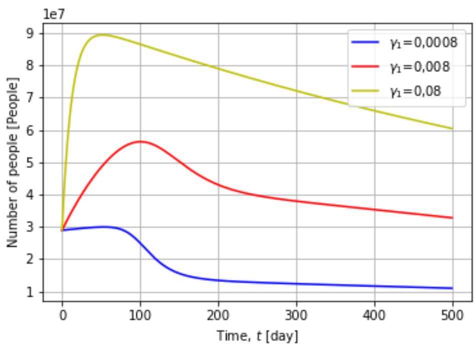

Figure 9. Number of people in vaccine compartment 1

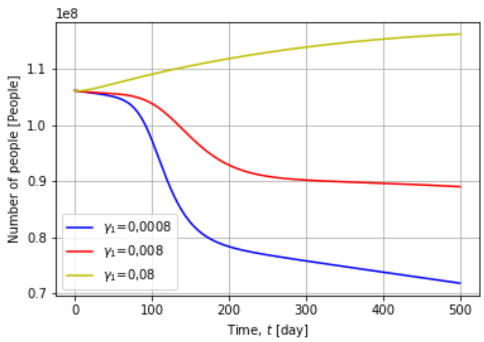

Figure 10. Number of people in vaccine compartment 2

Figure 9 presents the dynamics of the vaccinated population 1. The value of the vaccination rate parameter used is 0.0008; 0.008; 0.08 per day; from this figure, the higher the parameter value, the more population is in vaccine compartment one (1). The same trend also occurred in the two-compartment vaccine (2). For a vaccination rate of 0.0008, the population increase in compartment one only occurs in around 90 days, and the increase is insignificant. The population in this compartment continues to decline due to the second vaccination phase. This phenomenon also occurs at vaccination rates of 0.008 and 0.08, but what is different is that at these vaccination rates, the initial increase in population is quite significant. Furthermore, looking at Figures 9 and 10, the population of Vaccine Compartment 1 is greater than that of Vaccine Compartment 2 because the second-stage vaccination rate is lower than the first-stage vaccination rate. This shows that not everyone has been vaccinated for stage 1 vaccine stage 2.

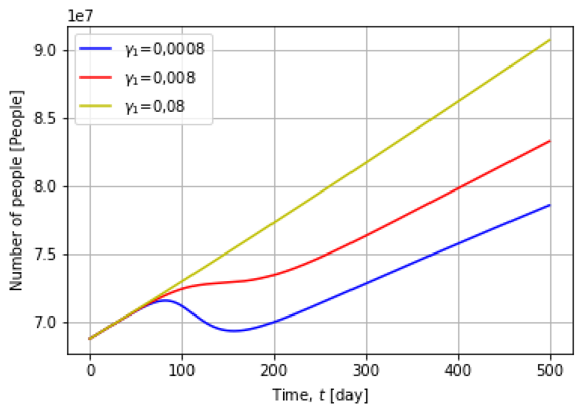

Figure 11. Number of people in vaccine compartment 3

The first vaccination rate changes also affect the population in vaccine compartment 3. The higher the first vaccination rate, the higher the number of people who are in vaccine three. This can be seen in Figure 11. Changes in the first vaccination rate also affect the number of people in the infected population.

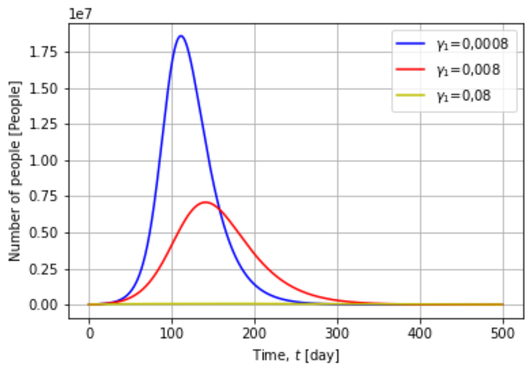

Figure 12. The number of people in the infected compartment

Figure 12 shows that the decrease in infected people is due to an increase in the first vaccination rate. The figure shows that the change in the first vaccination rate from 0.0008 to 0.008 drastically decreased the number of people infected. At the time of the first vaccination rate of 0.0008, the highest number of infected people was around 17.5 million and occurred around the 100th day. Then, the infection rate was increased to 0.008, and the maximum number of infected people was around 7 million, reaching around day 150. This change in the vaccination rate also affected the number of people recovering from COVID-19 (Recovery).

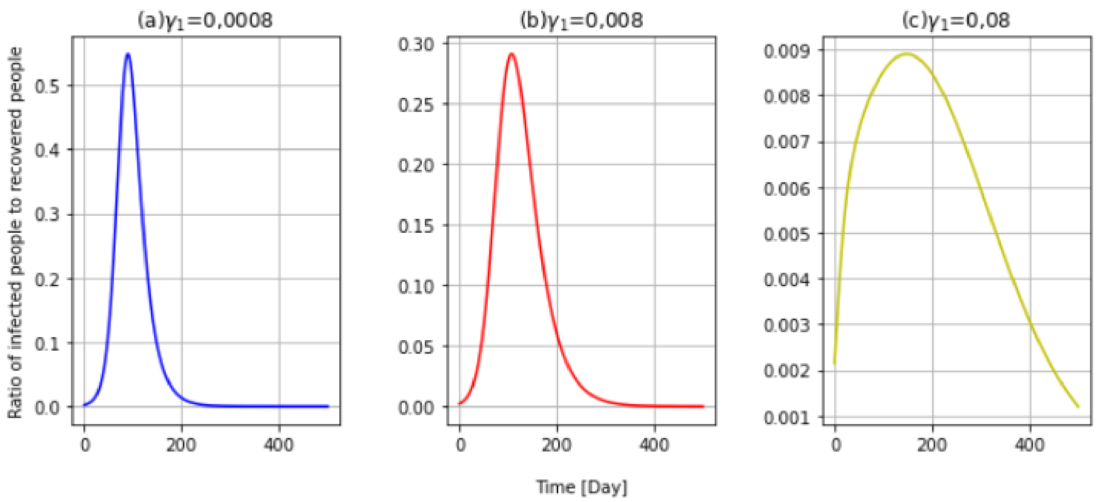

Figure 13. The ratio between the number of infected people and those who recovered

Figure 13 shows the ratio between people infected with COVID-19 and people who recovered from COVID-19 based on the difference in the rate of first vaccination. From this figure, the higher the rate of first vaccination, the smaller the ratio between infected and recovered people. This case means that the higher the rate of first vaccination, the more people who recover from being infected with Covid-19 will increase. In other words, if the rate of first vaccination is increased, it will speed up the recovery of people affected by Covid-19.

Figure 14 shows the ratio of infected people compared to people who died. As seen in the figure, when the rate of first vaccination increases, the ratio between infected people and those who die decreases, and the value is more than one (1), except for the rate of first vaccination, which is 0.08. It happening indicates that the number of people who died from COVID-19 is less than the number of people who were infected.

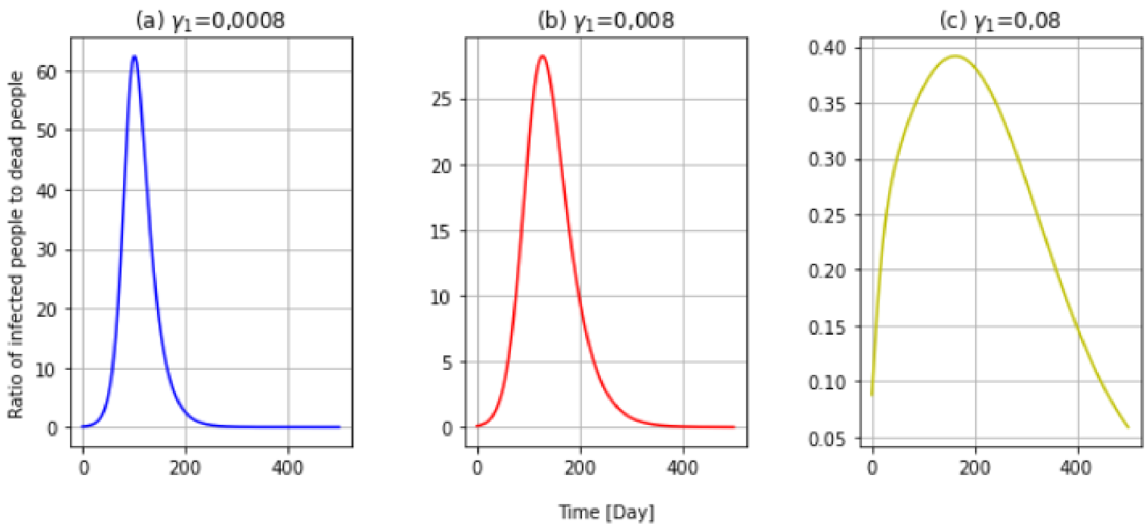

Figure 14. The ratio between the number of infected people and those who dead

Furthermore, the sensitivity of the second phase vaccination rate was also carried out. The purpose of the sensitivity analysis of this parameter is to see changes in infected people if there is a change in the number of vaccinated people in the second phase. The data used to simulate this stage is in Table 2, with the stage 2 vaccination rate being varied based on the data in the table below.

Sensitivity of vaccination parameters, γ_2

Symbol	Value	*Source*	*unit*
γ_2	0.0009	assumed	1/day
γ_2	0.009	assumed	1/day
γ_2	0.09	assumed	1/day

The simulation results of the sensitivity analysis of the second phase vaccination rate are presented in Figures 15, 16, and 17. Figure 15 shows the dynamics of the number of people infected with Covid-19 occasionally. The higher the stage two vaccination rate value, the lower the maximum number of people infected. When the second vaccination rate is 0.0009 per unit of time, the maximum number of infected people is around 17.5 million, which occurs around day 110. For a vaccination rate of 0.009 per unit of time, the maximum number of infected people is around 15.5 million on around day 110. Furthermore, around day 115, for a vaccination rate 0.09 per unit time, the peak of COVID-19 sufferers was around 12.5 million.

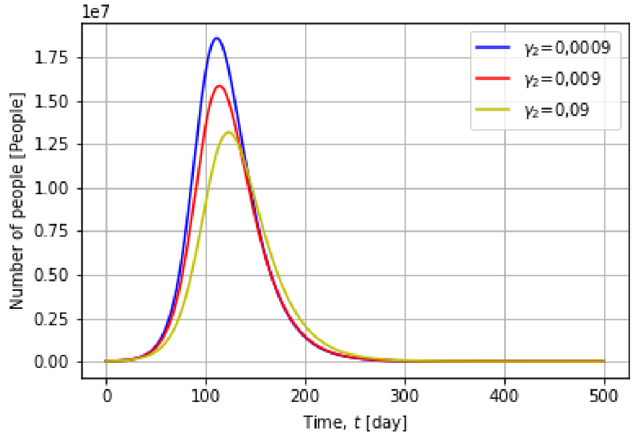

Figure 15. The number of people who have the infection

In the first 200 days, the number of people in the compartment recovering from infection experienced a drastic increase, after which it stabilized at a certain number. As mentioned earlier, the higher the second vaccination rate, the fewer infected people. This affects the number of people recovering from COVID-19, namely, fewer and fewer people recovering from COVID-19.

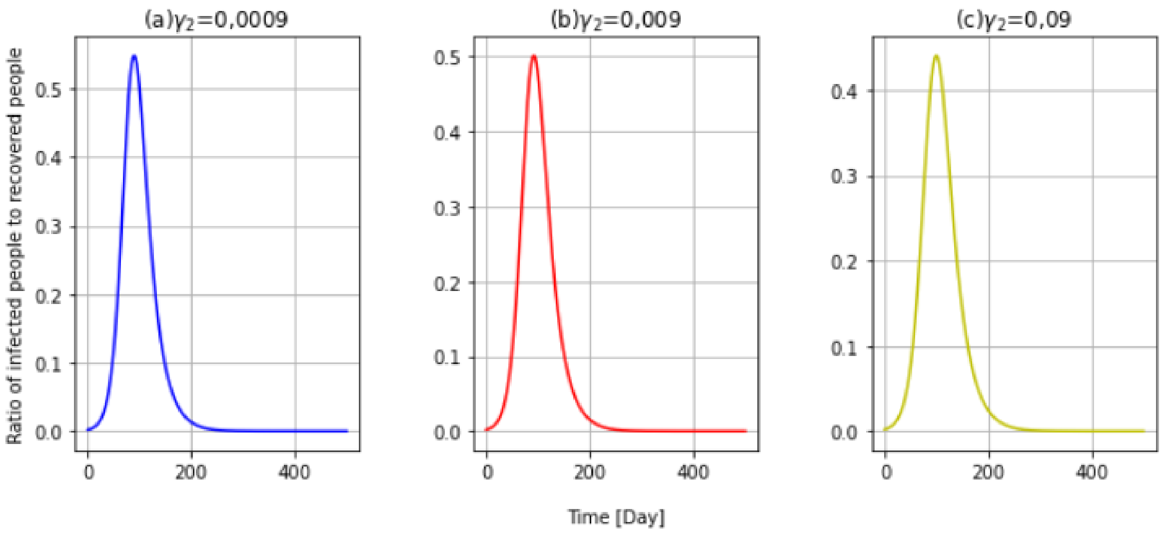

Figure 16. The ratio between the number of infected people and those who recovered

The effect of the rate of second vaccination on the ratio between people who were infected and people who recovered is presented in Figure 16. In this figure, the higher the rate of second vaccination, the lower the ratio. This case means that the rate of second vaccination affects the speed of recovery of people infected with COVID-19. As seen in Figure 16, the ratio value is less than one (1), meaning that the speed of people recovering will increase as the rate of second vaccination increases.

Furthermore, the positive impact of increasing the second phase vaccination rate can also be seen from the ratio between infected people and dead people, which is presented in Figure 17. In this figure, the ratio between people who infected and died is more than one (1). This phenomenon means that the number of people who die from COVID-19 is smaller than the number of people who are sick.

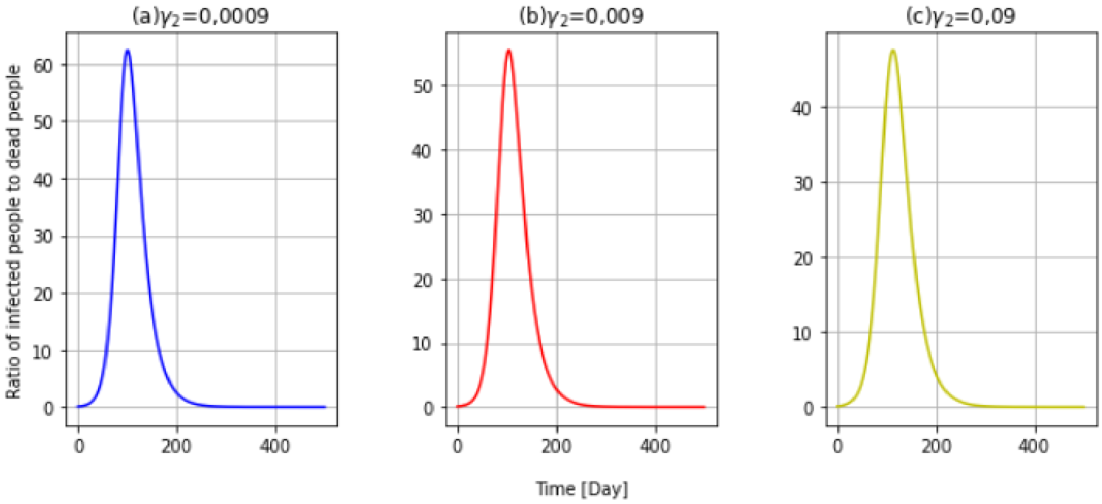

Figure 17. The ratio between the number of infected people and those who dead

The third sensitivity analysis of vaccination rates was carried out to see the effect of these parameters on the number of people infected. The third vaccination rate data used for Sensitivity of vaccination parameters, γ_3 sensitivity analysis is presented in the table below. The other parameter data uses the same data in Table 2.

Symbol	Value	*Source*	Unit
γ_3	0.0004	assumed	1/day
γ_3	0.004	assumed	1/day
γ_3	0.04	assumed	1/day

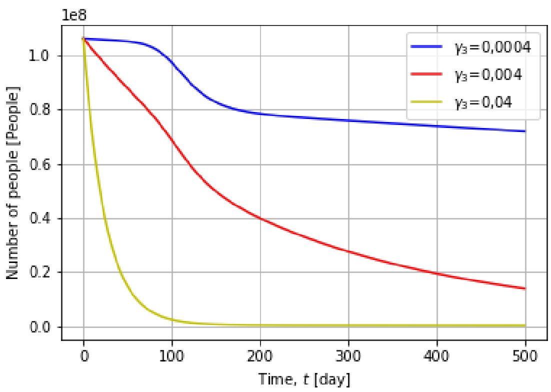

Figure 18. Number of people in vaccine compartment 2

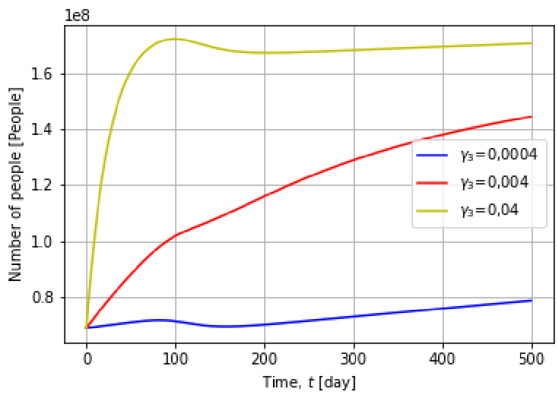

Figure 19. Number of people in vaccine compartment 3

The sensitivity analysis results of the third vaccination rate for the number of people in the second and third vaccination compartments are presented in Figures 18 and 19. The figure shows that the higher the third vaccination rate, the fewer people in the second vaccine compartment. Many people cause this vaccination the second to continue the third vaccine. As a result, the number of people in the third vaccine compartment also increases as the third vaccination rate increases.

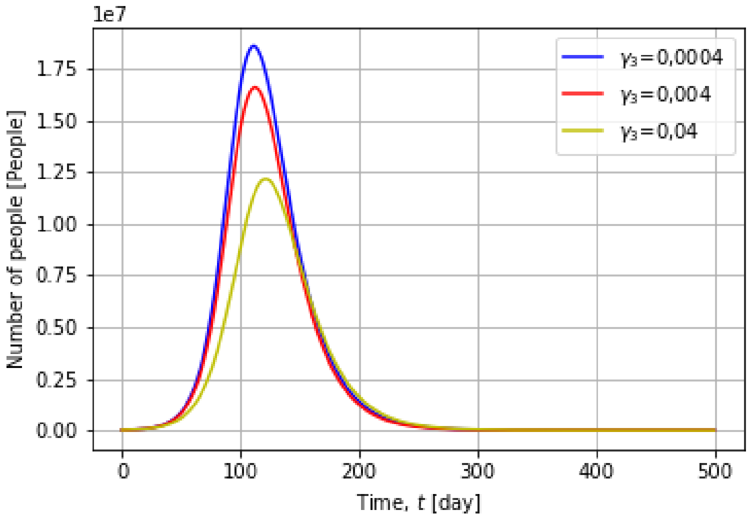

Figure 20. Number of people infected with COVID-19

Figure 20 shows changes in the number of infected people daily, with a change in the third vaccination rate. The figure shows that the higher the third vaccination rate, the fewer people infected with COVID-19. If more and more people take the vaccine up to stage three, the number of people infected with Covid will decrease.

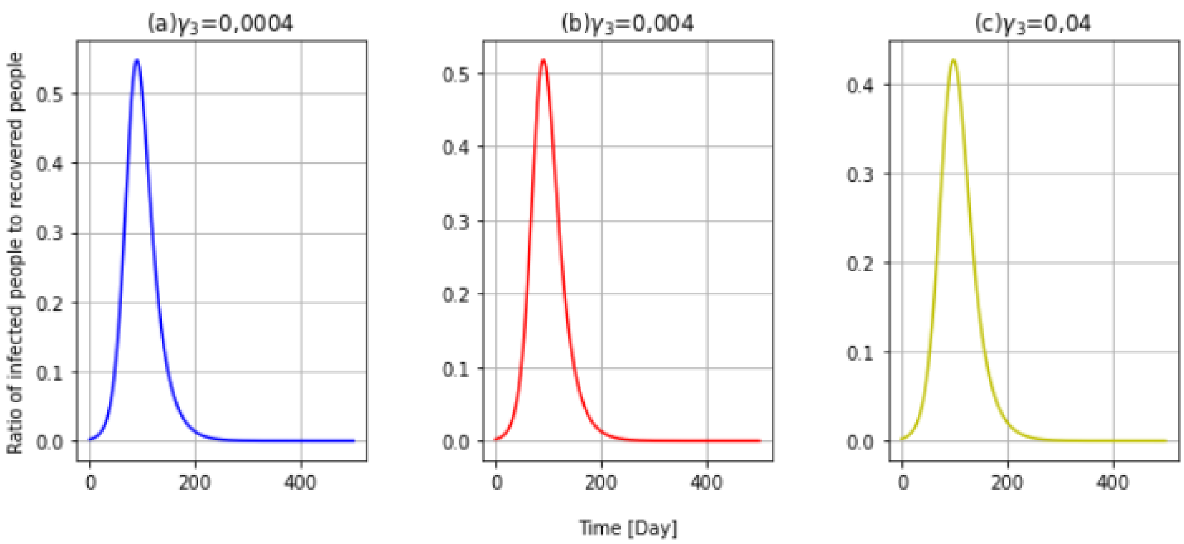

Figure 21. The ratio between the number of infected people and those who recovered

The effect of changing the third vaccination rate on the ratio between infected and recovered people is presented in Figure 21. In this figure, the behaviour is the same as the previous change in vaccination rate. In general, for the data and cases simulated using the model in this study, increasing the vaccination rate at each level can speed up recovery for infected people.

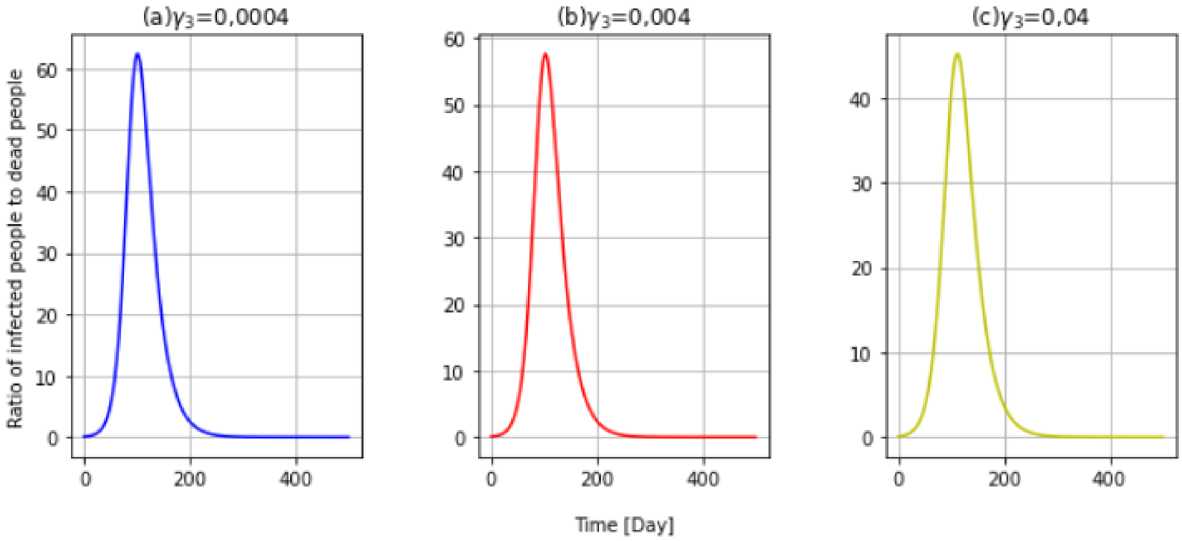

Figure 22. The ratio between the number of infected people and those who dead

Figure 22 is a picture of the ratio between people infected and people who died due to Covid disease. From this figure, the third vaccination rate results in a ratio of more than one person infected with Covid 19 to one who died. This occurrence shows that fewer people have died from Covid-19 than those infected with Covid-19.'

From the results of model simulations with several case studies/simulation schemes carried out in this research, it appears that vaccination from each stage of vaccination can reduce cases of COVID-19 infection. In addition, increasing the vaccination rate at each stage of vaccination can speed up the number of people affected. COVID-19 infection is cured.

Limitations

This research development opportunity is still quite a lot. The data uses secondary data. Apart from that, validation of the model has also yet to be carried out; only numerical simulations have been carried out. Furthermore, this model still needs to analyze the equilibrium solution and its stability.

Conclusion

A model of the spread of COVID-19 that considers the three stages of vaccination has been carried out. The model was developed from the basic SIR model. The model was solved numerically, and a sensitivity analysis was carried out. From the numerical simulations, it can be concluded that vaccination can reduce cases of Covid-19 infection. The first, second, and third vaccination rates show the same trend; the higher the vaccination rate, the fewer people will be infected with COVID-19. Please note that the data used for simulations in this study, especially for the vaccination rate at each stage, is assumed data. From the simulation results, the lowest number of people were affected by COVID-19 when using assumed data, with a first vaccination rate of 0.08 per day, a second vaccination rate of 0.0009 per day, and a third vaccination rate of 0.0004 per day. Apart from that, from the simulations, the increasing vaccination rate at each vaccine stage can speed up people's recovery from COVID-19 infection.

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