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Mathematical Modelling in Energy Measurement Production of Water Tap Flow

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Abstracts

Electric power is vital in supporting the economy of current society. Electric power can be obtained from water. Water is one of renewable energy that can be used as hydropower to produce electrical energy. The electric energy can be produced from a generator driven by a waterwheel, which means the electric energy comes from the kinetic energy of water. One of the most common sources for flowing water is from the water tap. The electric power that could be produced from flowing water with a restriction that the water flow only affected by gravitational force and come from a finite resource. In this research, a mathematical model of electric energy or electric power are developed based on kinetic energy. The change of kinetic energy with time is electric power. The mathematical model is solved numerically to find kinetic energy using the Euler method; therefore, this model is able to obtain power with changes in time. The result test show that the power greatly affected by the volume and height of water. The volume and height of water are decreasing over time and also the kinetic energy and power produced from water flowing out from the water tap reduced exponentially with respect to time.

Keywords: Electricity, kinetic energy, mathematical modelling, power, water

Introduction

Indonesia is a country that has a large area and a high population. As a country with diverse geography and a dispersed population, Indonesia still faces the challenge of supplying energy to all its citizens. In 2016, 7 million households, or about 28 million Indonesians, still had no electricity access. Renewable energy potential from Indonesia for electricity reaches 443 GW, including geothermal, micro-mini hydro, wind, ocean wave, and solar bioenergy. The potential of solar power in Indonesia has the largest portion, more than 207 MW, followed by water and wind [1].

With Indonesia's mountainous and hilly topographic conditions, Indonesia has a bounty of water energy potential. This potential is a government priority to achieve a new and renewable energy mix of at least 23% by 2025 and at least 31 by 2030. To achieve this target, the government determines the direction of policies and strategic plans listed in the Rencana Umum Energi Nasional (RUEN). Water energy can be used according to the amount of power generated, namely PLTA, PLTM, PLTMH, and energy storage with pumped storage technology [2].

Water is a low-cost and relatively easy source of energy because water stores potential energy (falling water) and kinetic energy (flowing water). The point obtained from flowing water is called Hydroelectric power. Water's energy can be utilized and used in mechanical and electrical power. Water energy is widely used by waterwheels or water turbines that harness the presence of a waterfall or the flow of water in the rivers [3]. Like other substances, water has two kinds of energy, which are kinetic and potential energy. The kinetic energy in water is the main energy and is the energy for action and motion. Kinetic energy is the reason why water can flow wave existed. Meanwhile, the potential of energy is stored in the water. This potential energy becomes useful when the water starts flowing, where it will be transferred into kinetic energy [4] [5].

The need for electric power is a very vital thing in supporting the economy of a society. The reason is easy to understand. Electricity is the driving energy of motors or machines in industrial processes. Moreover, the role of electricity in this modern era was increasing much more. It takes effort to get electricity easily and cheaply for remote areas or those that have not been reached by PLN electricity. This should be done independently by utilizing the natural potential around the location. According to previous research, cheap power plants can be obtained with solar steam or water energy [6]. Therefore, we took a topic in which they made a prototype of a water tap flow. Then we made modeling from that prototype, with data variables and the shape of a container supported by the calculation of numerical solutions.

The flow rate of water is a unit to approach the hydrological values of processes that occur in the field. In a watershed area, the ability to measure the flow rate is required to determine the potential of water resources. To observe and evaluate the water balance of an area can use flow rate through the approach of surface water resource potential [7].

Measurement of flow rate water can be done by measuring the speed of water flow in a container with a cross-sectional area of a certain size. Several methods can be used to measure the velocity of water flow in rivers or grooves: The area-velocity method, the Slope area method, the Tracer method, Weir and Flume, and the Volumetric method area. The velocity can be measured by methods: the current-meter method and the floating method. Then the velocity distribution in the groove is not the same in the horizontal or vertical direction [8]. Currently, many flow rate water measurement tools are available on the market. Still, most are provided for measuring flow rate in a cross-section of pipes because the velocity of water is a parameter that can represent the amount of water discharge [9], i.e., multiplying it by the cross-sectional factor of the measuring area.

In this study, we try to find the mathematical model for a water flow from a tank using a tap source to see how much energy can be generated. A water tap is an object that is always found in daily life and provides a clean flow of water. But in addition to running water, running water that taps can also be used for other things, such as to produce electrical energy.

The limitations of the problems we take based on the main reference sources in the manufacture of hydroelectric miniature on flat current to move the alternator include:

1) Manufacture of a waterwheel
2) Working principles of the waterwheel
3) Function of waterwheel components

We also find a solution modelling differential equations of energy generated from the flow of water from the tap and find out the energy generated that can be beneficial if utilized. In the process of those waterwheel principles, it will seek how to create the mathematical modelling in calculating the energy and electrical power generated from the container's water tap flow.

Material and Methods

In the calculation, this research used the International System of Units (SI) with Meter Kilo Second (MKS) as base units [10].

Table 1. Variable and Parameter for Model

Variable/Parameter	Description	Unit
E_k	Kinetic energy	$Joule$ or $kg\ m^2/s^2$
A	Cross-sectional area of tank	m^2
V	Water volume of tank	m^3

Table 1(continuous). Variable and Parameter for Model

Variable/Parameter	Description	Unit
a	Area of tap water's hole	m^2
v	Water velocity	m/s
ρ	Density of water	kg/m^3
m	Mass of water	kg
h_0	Initial height of water in tank	m
h	Height of water in tank	m
g	Gravitational acceleration	m/s^2
t	Time	s
P	Power	$Watt$ or $kg\ m^2/s^3$
Q	Flow rate of water	m^3/s

In this research, the model developed based on kinetic energy is [11]:

$$E_k = \frac{1}{2}mv^2 \quad (1)$$

where $v = \sqrt{2gh}$, so that

$$E_k = \frac{1}{2}m\left(\sqrt{2gh}\right)^2 \quad (2)$$

The rate of kinetic energy is equal to power [7], $P = \frac{dE_k}{dt}$, so that

$$\frac{dE_k}{dt} = \frac{d}{dt}(V\rho gh) \quad (3)$$

In this case density of water (ρ) gravitational acceleration (g), and water volume of the tank also the height of the water in the tank changes with the time when the faucet is opened, then

$$\frac{dE_k}{dt} = \rho gh\frac{dV}{dt} + \rho gV\frac{dh}{dt} \quad (4)$$

Because $\frac{dV}{dt} = Q$, then

$$\frac{dE_k}{dt} = \rho ghQ + \rho gV\frac{dh}{dt} \quad (5)$$

So that

$$P = \rho ghQ + \rho gV\frac{dh}{dt} \quad (6)$$

and

$$P = \rho ghQ + \rho gV\left(-\frac{a}{A}\sqrt{2gh}\right) \quad (7)$$

Assumed volume of water is decreased, so that

$$P = \rho ghQ + \rho g(-V)\left(-\frac{a}{A}\sqrt{2gh}\right) \quad (8)$$

or

$$P = \rho ghQ + \rho g(V)\left(\frac{a}{A}\sqrt{2gh}\right) \quad (9)$$

Based on the development for the formula obtained above in the form of energy values and constant power, it does not produce enough reality results in accordance with the main literature study; this is due to the high volume of water in the tank that decreases according to the volume of water coming out.

Therefore, the equation model of the development needs to be developed again to get the value of height changes of water in the tank [12].

$$A\frac{dh}{dt} = -av \quad (10)$$

or

$$\frac{dh}{dt} = \frac{-a}{A}\sqrt{2gh}, \quad (11)$$

With initial condition $h(0) = h_0$, so that the analytic solution for this equation is [12]

$$h = \left(\sqrt{h_0} - \frac{at\sqrt{2g}}{2A}\right)^2 \quad (12)$$

The amount of water coming out of the tank is the velocity of water flow multiplied by the area of the tap water's hole. The result is equivalent to the flow rate of water change in the tank multiplied by the cross-sectional area. Other parameters will be updated using the height change (h) from the model we obtained.

That has been a model for energy and power that accepts changes in the height of water level. Then because the power is the rate of energy with respect to time, the energy's equation model will be redeveloped using the rate that has been obtained before.

The model that has been obtained still have a shortcoming, where the value of the rate of energy that respect to time always changes over time. Meanwhile, the model only works if the value of that rate is constant. Therefore, in the calculation we applied Euler method in our modelling.

Results and Discussion

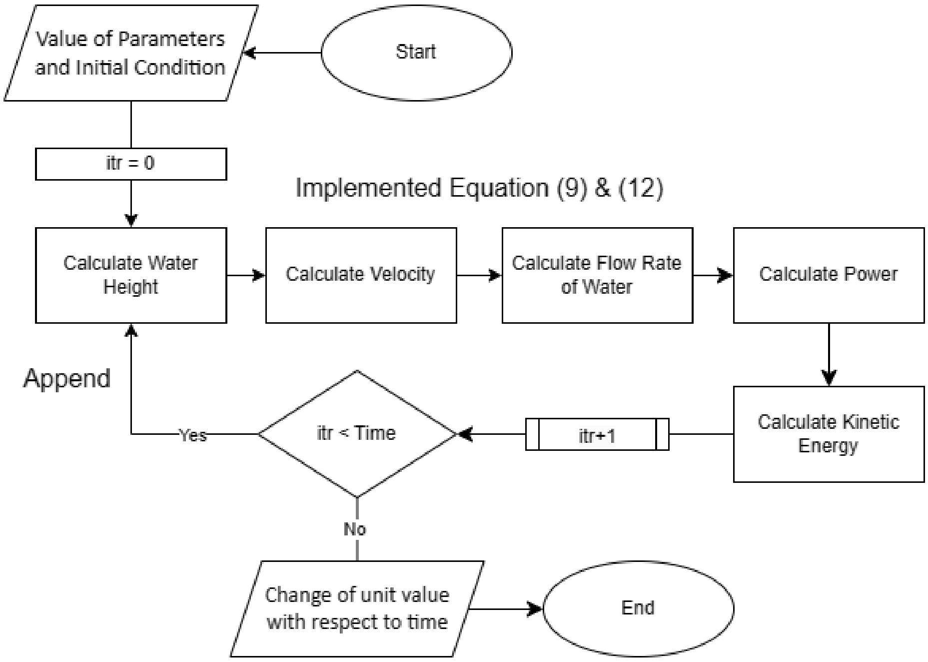

Figure 1. Flowchart of Calculation Process

In this section, we do the calculation in the program with the process that figure out in Figure 1. The model will be tested using a set of initial data that can be seen in Table 2 and derived data from base equations in Table below.

Table 2. Value of Variable and Parameter for Numerical Simulation

Variable/Parameter	Description	Value	Unit
A	Cross-sectional area of tank	0.2827	m^2
V	Water volume of tank	0.4948	m^3
a	Area of tap water's hole	0.0001	m^2
v	Water velocity	5.8596	m/s
ρ	Density of water	1000	kg/m^3
m	Mass of water	494.8008	kg
h_0	Initial height of water in tank	1.75	m
g	Gravitational acceleration	9.81	m/s^2
t	Time	459	s
Q	Flow rate of water	0.0007	m^3/s

The test is done by using our model, which has been extended to apply Euler method in Figure 2 until Figure 6, which will show change in height, velocity, flow rate, kinetic energy, and power from the tap water system that only depend on gravity for a given time. Time used to test this model is 450 second, where the testing result will be shown as a graph of change in respect to time.

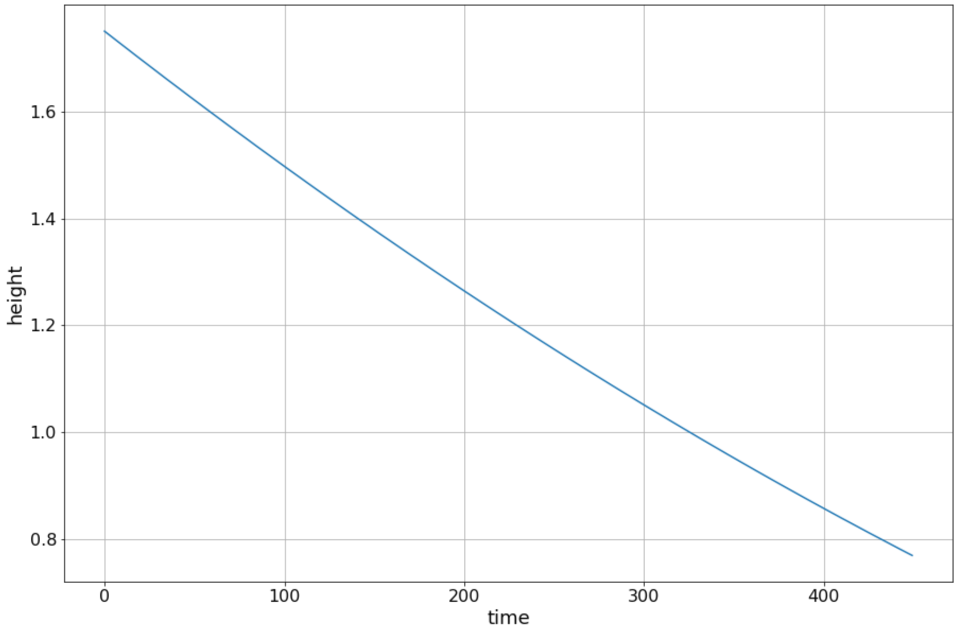

Figure 2. Change of Height in Respect to Time (s)

Figure 2 is shown change of height in respect to time. The height of water is always decreasing anytime because water in the tank is flow out from tank. Unit of height in meters and time in second. Initial value the water height about 1.7 meters and about 450 minutes became about 0.8 meters.

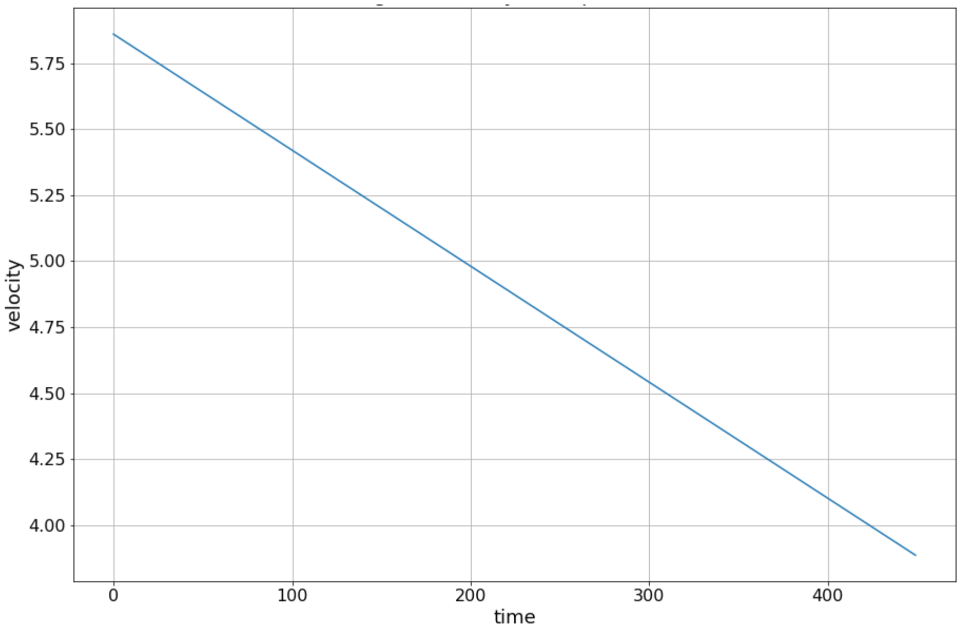

Figure 3. Change of Velocity in Respect to Time (s)

The velocity of water anytime is shown in Figure 3. From this figure it appears that velocity always decreasing. At the beginning velocity of water about 5.75 meters per second, after 450 minutes velocity become about 4 meters per second.

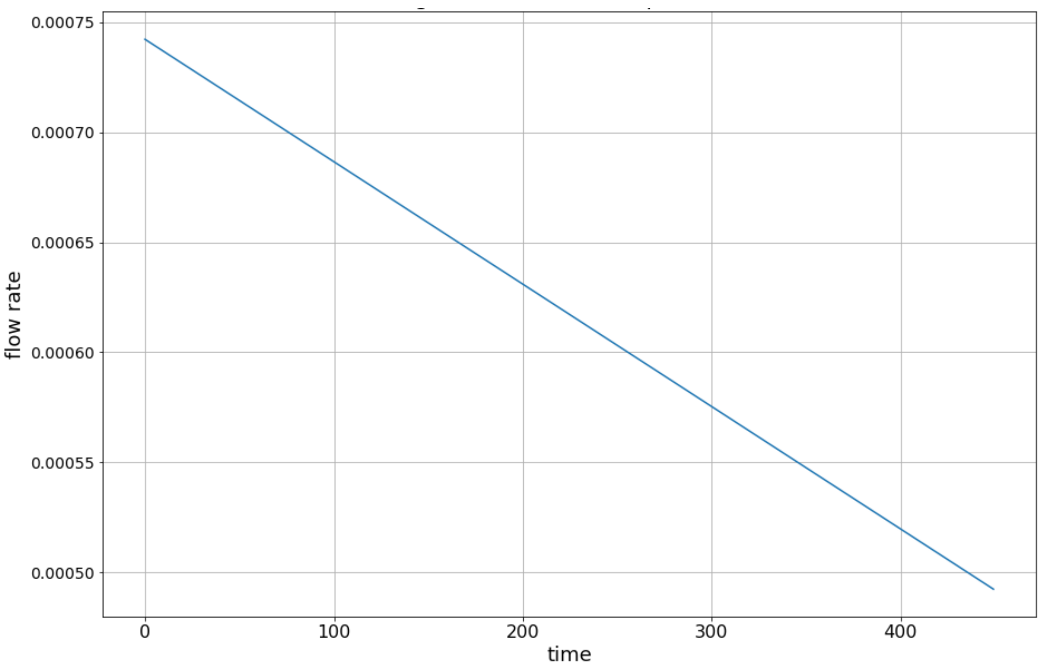

Figure 4. Change of Flow Rate in Respect to Time (s)

Furthermore, change of flowrate in respect to time is shown in Figure 4 and change of kinetic energy in respect to time is shown in Figure 5. Same as the previous figure, behavior of flowrate and kinetic energy are also decreasing anytime. For the rate of kinetic energy, the kinetic energy is rapidly descended until halfway time mark, where the change rate between kinetic energy in each second become smaller and maybe stopped all together at some time.

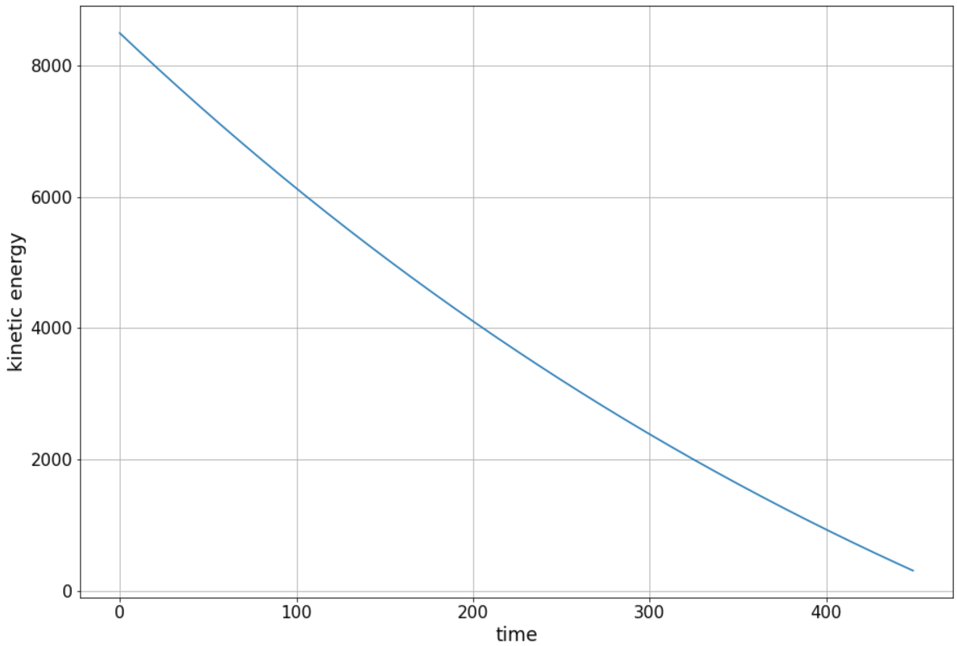

Figure 5. Change of Kinetic Energy in Respect to Time (s)

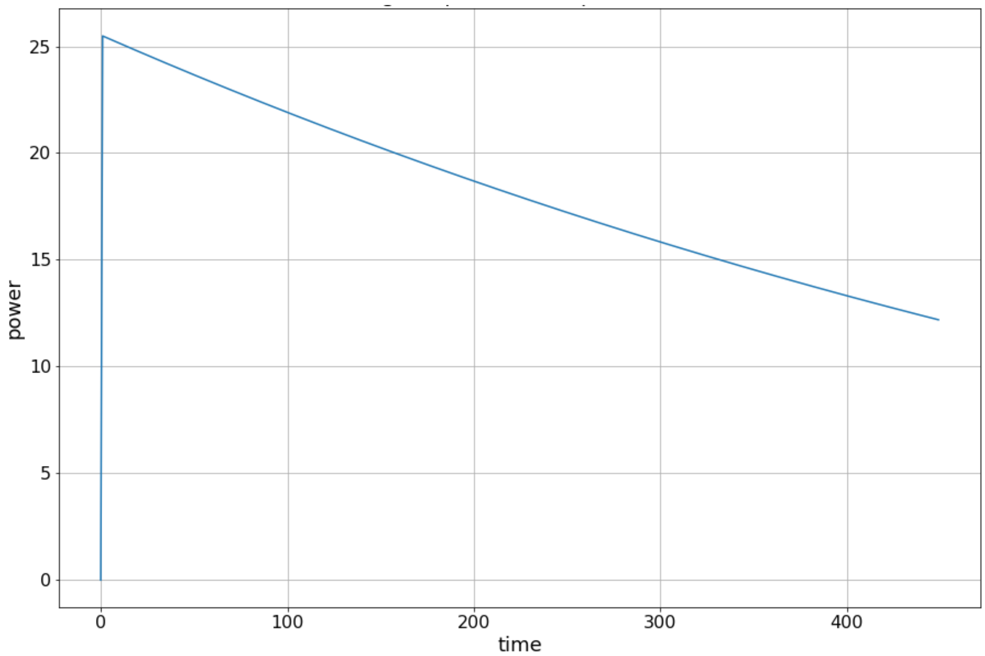

Figure 6. Change of Power in Respect to Time (s)

Similar to kinetic energy, the rate change of power with respect to time also becomes smaller after the halfway mark. However, if attention is at the beginning, the power suddenly rises; this is because the water faucet has just been opened, and the volume is still high, so the initial zero power suddenly becomes high.

Conclusion

From all of the solutions result above, it can be concluded that the downfall of water that is affected by the gravitational gravity and elevation will produce the velocity of flow water, the value of change in kinetic energy, water velocity, and water flow rate decreases over unit time as water level height decreases. the power generated at the beginning (initial state) produces the maximum value and decreases over time.

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